

**THE DEGREE OF BINDING RESULTING FROM REACTION  
POINTS EXPRESSED IN A DIATOPIC TABLE.  
AN APPLICATION TO A CATALAN VERB MORPHOLOGY  
DATABASE**

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## **0. Introduction**

Antoni M. Alcover: 500,000 verb forms corresponding to the complete conjugation of 75 verbs in 149 localities in the Catalan-speaking territory.

“La flexió verbal en el dialectes catalans” (1929 – 1933).

### **1. Techniques of Integration**

### **2. Assessing the degree of integration**

### **3. Geolinguistic interpretation**

### **4. Conclusion**

# 1. Techniques of Integration

Form	1	2	3	4	5	6	7	8	9	10
cantar-1-cant										
cantar-1-canta										
cantar-1-cante										
cantar-1-canti	V	V	V	V	V	V	V	V	V	V
cantar-1-cantic										
cantar-1-canto										
cantar-1-cantoc										
cantar-1-cantot										
cantar-2-cantas	V	V	V	V	V	V	V	V	V	V
cantar-2-cantes	V	V	V	V	V	V	V	V	V	V
cantar-3-canta				V			V			V
cantar-4-cantam	V	V	V		V	V		V	V	V
cantar-5-cantau				V			V			V
cantar-5-canteu	V	V	V		V	V		V	V	V
cantar-6-canten	V	V	V	V	V	V	V	V	V	V

Table 1.

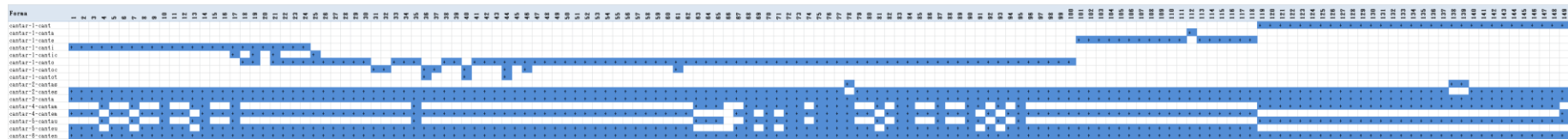


Figure 1. Initial data matrix

## 1.1 Average distance with respect to the zero point (ADZP)

Minkowski distance:

$$\text{ADZP} = [ (\sum_i x_i h_i^N) / \sum_i x_i ]^{1/N}$$

where  $x_i$  is 1 if there is reaction in form of “v”; or zero (0) if there is no reaction;  $h_i$  is the sequential number (1, 2, ..., p) counting from the point of origin (0).

D	L-1	L-2	L-3	L-4	Dist
d-1	v	v			1.581
d-2			v		3.000
d-3		v			2.000
d-4			v	v	3.536
d-5	v	v	v		2.160
Dist	3.606	3.416	3.873	4.000	

Table 2.

$$\text{d-1: } [(1^2 + 2^2) / 2]^{1/2} = 1.581$$

$$\text{d-2: } [(3^2) / 1]^{1/2} = 3.000$$

$$\text{d-3: } [(2^2) / 1]^{1/2} = 2.000$$

$$\text{d-4: } [(3^2 + 4^2) / 2]^{1/2} = 3.535$$

$$\text{d-5: } [(1^2 + 2^2 + 3^2) / 3]^{1/2} = 2.160$$

$$\text{L-1: } [(1^2 + 5^2) / 2] = 3.606$$

$$\text{L-2: } [(1^2 + 3^2 + 5^2) / 3] = 3.416$$

$$\text{L-3: } [(2^2 + 4^2 + 5^2) / 3] = 3.873$$

$$\text{L-4: } [(4^2) / 1] = 4.000$$

D	L-1	L-2	L-3	L-4
d-1	v	v		
d-2			v	
d-3		v		
d-4			v	v
d-5	v	v	v	

→

D	L-1	L-2	L-3	L-4
d-1	v	v		
d-3		v		
d-5	v	v	v	
d-2			v	
d-4			v	v

Table 3.

Lv	L-1	L-2	L-3	L-4
d-1	v	v		
d-2			v	
d-3		v		
d-4			v	v
d-5	v	v	v	

→

LL	L-2	L-1	L-3	L-4
d-3	v			
d-1	v	v		
d-5	v	v	v	
d-2			v	
d-4			v	v

Table 4.

$$d-1: [(1^2 + 2^2) / 2]^{1/2} = 1.581 \quad (...1)$$

$$d-2: [(3^2) / 1]^{1/2} = 3.000 \quad (...4)$$

$$d-3: [(2^2) / 1]^{1/2} = 2.000 \quad (...2)$$

$$\text{d-4: } [(3^2 + 4^2) / 2]^{1/2} = 3.535 \quad (\dots 5)$$

$$\text{d-5: } [(1^2 + 2^2 + 3^2) / 3]^{1/2} = 2.160 \quad (\dots 3)$$

Lv	L-1	L-2	L-3	L-4	Value
d-1	v	v			1.581
d-3		v			2.000
d-5	v	v	v		2.160
d-2			v		3.000
d-4			v	v	3.536

Table 5.

$$\text{L-1: } [(1^2 + 3^2) / 2]^{1/2} = 2.236 \quad (\dots 2)$$

$$\text{L-2: } [(1^2 + 2^2 + 3^2) / 3]^{1/2} = 2.160 \quad (\dots 1)$$

$$\text{L-3: } [(3^2 + 4^2 + 5^2) / 3]^{1/2} = 4.082 \quad (\dots 3)$$

$$\text{L-4: } [(5^2) / 1]^{1/2} = 5.000 \quad (\dots 4)$$

Lv	L-2	L-1	L-3	L-4	Value
d-1	v	v			1.581
d-3	v				1.000
d-5	v	v	v		2.160
d-2			v		3.000
d-4			v	v	3.536
Value	2.160	2.236	4.082	5.000	

Table 6.

$$d-1: [(1^2 + 2^2) / 2]^{1/2} = 1.581 \quad (...2)$$

$$d-3: [(1^2) / 1]^{1/2} = 1.000 \quad (...1)$$

$$d-5: [(1^2 + 2^2 + 3^2) / 3]^{1/2} = 2.160 \quad (...3)$$

$$d-2: [(3^2) / 1]^{1/2} = 3.000 \quad (...4)$$

$$d-4: [(3^2 + 4^2) / 2]^{1/2} = 3.535 \quad (...5)$$



Lv	L-2	L-1	L-3	L-4	Value
d-3	v				1.000
d-1	v	v			1.581
d-5	v	v	v		2.160
d-2			v		3.000
d-4			v	v	3.536
Value	2.160	2.550	4.082	5.000	

Table 7.

$$L-2: [(1^2 + 2^2 + 3^2) / 3]^{1/2} = 2.160 \quad (...1)$$

$$L-1: [(2^2 + 3^2) / 2]^{1/2} = 2.550 \quad (...2)$$

$$L-3: [(3^2 + 4^2 + 5^2) / 3]^{1/2} = 4.082 \quad (...3)$$

$$L-4: [(5^2) / 1]^{1/2} = 5 \quad (...4)$$

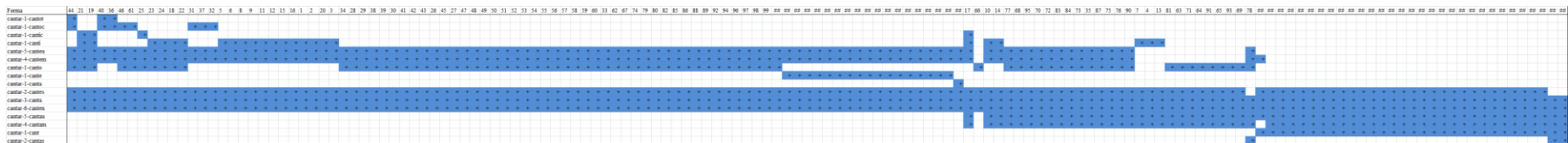


Figure 2. Integration by the distance regarding the point of origin

## 1.2. Other multivariate methods

### 1.2.1 Correlation coefficient matrix

P1	v-1	v-2	v-3	v-4		P1	v-2	v-1	v-3	v-4
v-1	1.000	0.667	0.167	0.408		v-2	1.000	0.667	0.667	0.612
v-2	0.667	1.000	0.667	0.612		v-1	0.667	1.000	0.167	0.408
v-3	0.167	0.667	1.000	0.408		v-3	0.667	0.167	1.000	0.408
v-4	0.408	0.612	0.408	1.000		v-4	0.612	0.408	0.408	1.000

Table 8

P1	d-1	d-2	d-3	d-4	d-5		P1	d-4	d-3	d-1	d-2	d-5
d-1	1.000	0.577	0.577	1.000	0.577		d-4	1.000	0.577	1.000	0.577	0.577
d-2	0.577	1.000	0.333	0.577	0.333		d-3	0.577	1.000	0.577	0.333	0.333
d-3	0.577	0.333	1.000	0.577	0.333		d-1	1.000	0.577	1.000	0.577	0.577
d-4	1.000	0.577	0.577	1.000	0.577		d-2	0.577	0.333	0.577	1.000	0.333
d-5	0.577	0.333	0.333	0.577	1.000		d-5	0.577	0.333	0.577	0.333	1.000

Table 9.

P1	L-2	L-1	L-3	L-4
d-4			v	v
d-3	v			
d-1	v	v		
d-2			v	
d-5	v	v	v	

Table 10.

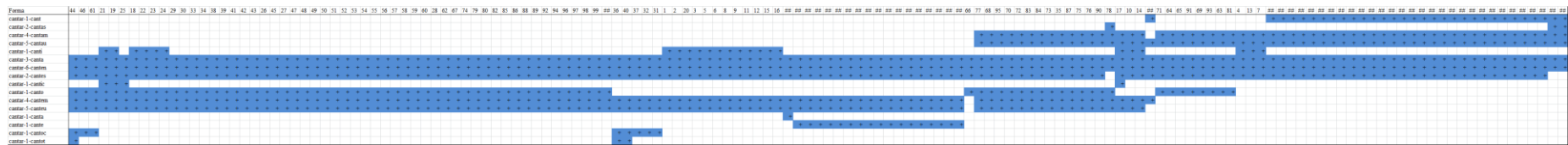


Figure 3. Integration by Correlation Coefficient

## 1.2.2 Principal Component Analysis

P1	L-3	L-2	L-1	L-4
d-2	v			
d-5	v	v	v	
d-3		v		
d-1		v	v	
d-4	v			v

Table 11.

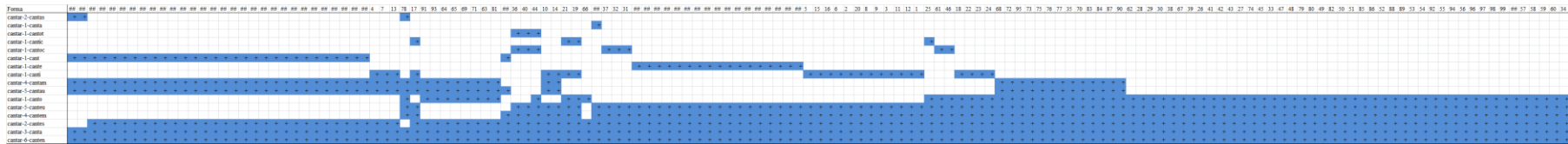


Figure 4. Integration by Principal Component Analysis

## 1.2.3 Factor Analysis

P1	L-3	L-4	L-1	L-2
d-2	v			
d-4	v	v		
d-1			v	v
d-5	v		v	v
d-3				v

Table 12.

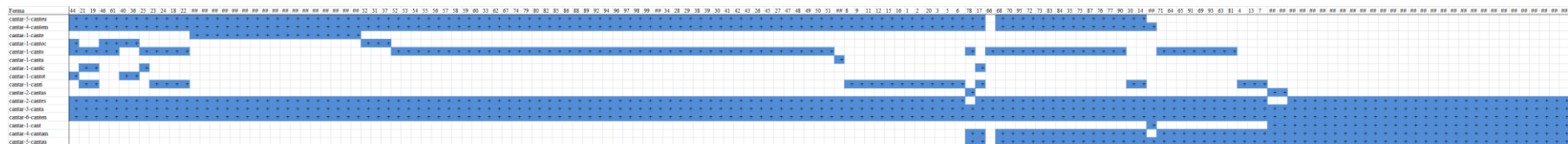


Figure 5. Integration by Factor Analysis

## 1.2.4 Quantification Type-III

P1	L-2	L-1	L-3	L-4
d-3	v			
d-1	v	v		
d-5	v	v	v	
d-2			v	
d-4			v	v

Table 13.

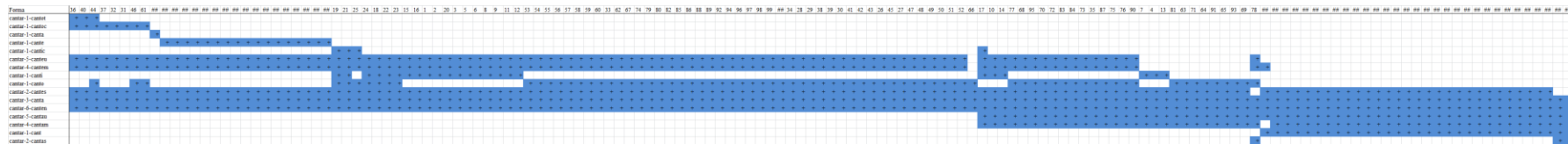


Figure 6. Quantification type-III

## 1.2.5 Cluster Analysis

P1	L-1	L-2	L-3	L-4
d-1	v	v		
d-3		v		
d-5	v	v	v	
d-2			v	
d-4			v	v

Table 14.

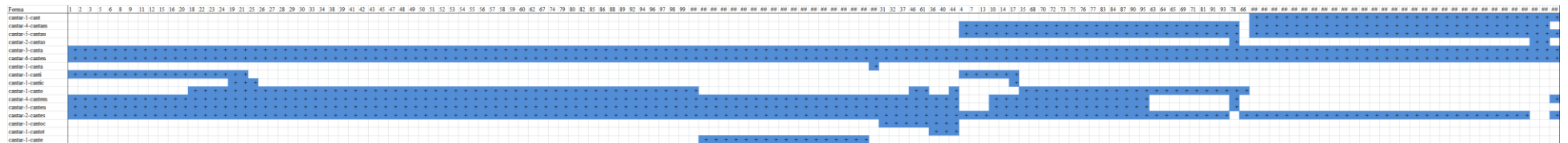


Figure 7. Integration by Cluster Analysis

## 2. Assessing the degree of integration

### 2.1. Sequential Average Distance (DMS)

$$\text{DMS} = \frac{\sum_i \sum_j \sum_a \sum_b [(i - a)^2 + (j - b)^2]^{1/2} |x_{ij} - x_{ab}|^{1/2}}{n}$$

where  $i$  and  $j$  are sequences of a point of the compared pair;  $a$  and  $b$  are the same as another point on the compared pair,  $n$  is the total number of compared pairs



## 2.2. Referential Average Distance (RAD)

$$\text{RAD} = \frac{\sum_i \sum_j \sum_a \sum_b [(\mathbf{v}_i - \mathbf{h}_a)^2 + (\mathbf{v}_j - \mathbf{h}_b)^2]^{1/2} |\mathbf{x}_{ij} \mathbf{x}_{ab}|^{1/2}}{n}$$

where  $\mathbf{v}$  y  $\mathbf{h}$  are vectors of vertical and horizontal references.

## 2.3. Sequential Correlation Coefficient (SCC)

Lv	L-1	L-2	L-3	L-4
d-1	v	v		
d-2			v	
d-3		v		
d-4			v	v
d-5	v	v	v	

→

Lv	L-2	L-1	L-3	L-4
d-3	v			
d-1	v	v		
d-5	v	v	v	
d-2			v	
d-4			v	v

Tables 15a and 15b.

Data :  $(X, Y) = (1, 1) (2, 1) (2, 2) (3, 1) (3, 2) (3, 3) (4, 3) (5, 3) (5, 4)$

SCC = 0.82

## 2.4. Referential Correlation Coefficient (RCC)

Lv	L-2	L-1	L-3	L-4	Value
d-3	v				-1.42
d-1	v	v			-0.71
d-5	v	v	v		-0.01
d-2			v		0.76
d-4			v	v	1.38
Value	-1.10	-0.82	0.58	1.34	

Table 16.

Data :  $(X, Y) = (-1,10, -1.42) (-1.10, -0.71) \dots (1.34, 1.38)$

$RCC = 0.84$

## 2.5. Mean Contact Index (MCI)

P1	L-1	L-2	L-3	L-4
d-1	v	v		
d-2		↔	v	
d-3		v		
d-4			v	v
d-5	v	v	v	↕

→

P1	L-2	L-1	L-3	L-4
d-3	v			
d-1	v	v		
d-5	v	v	v	
d-2			v	
d-4			v	v

Tables 17a and 17b.

P2	L-1	L-2	L-3	L-4
d-1	1	1	2	3
d-2	2	4	3	4
d-3	1	3	2	3
d-4	3	3	2	4
d-5	2	3	2	4

Table 18.

MCI: we divide the sum of links by the number of reactions.

## 2.6. Normal Contact Index (NCI)

$$\begin{array}{c}
 + - + - + \\
 | \\
 +
 \end{array}
 \quad
 \begin{array}{c}
 + - + \\
 | \\
 +
 \end{array}$$

**«U(3)= 2»**
**«U<sub>max</sub>(3)=2»**

When N = 4, U<sub>max</sub>(4) = 4 in the square distribution in the right-hand figure:

$$\begin{array}{c}
 + - + - + - + \\
 | \quad | \quad | \\
 + - +
 \end{array}
 \quad
 \begin{array}{c}
 + - + \\
 | \quad | \\
 + - +
 \end{array}$$

**«U(4)=3»**
**«U<sub>max</sub>(4)=4»**

$$\begin{array}{c}
 + - + - + \\
 | \quad | \\
 + - +
 \end{array}
 \quad
 \begin{array}{c}
 + - + - + \\
 | \quad | \quad | \\
 + - + - +
 \end{array}
 \quad
 \begin{array}{c}
 + - + - + - + \\
 | \quad | \quad | \\
 + - + - +
 \end{array}
 \quad
 \begin{array}{c}
 + - + - + - + \\
 | \quad | \quad | \quad | \\
 + - + - + - +
 \end{array}$$

**«U<sub>max</sub>(5)=5»**
**«U<sub>max</sub>(6)=7»**
**«U<sub>max</sub>(7)=8»**
**«U<sub>max</sub>(8)=10»**

```

+ - + - +
|   |   |
+ - + - +
|   |   |
+ - + - +

```

**«U<sub>max</sub>(9)=12»**

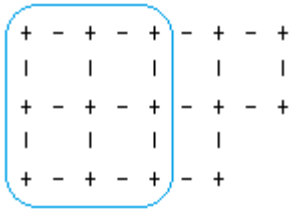
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+ - + - + - + - +
|   |   |   |   |
+ - + - + - + - +
|   |   |   |
+ - + - + - +

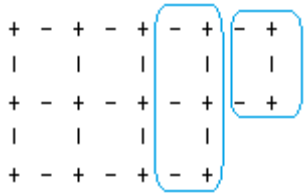
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**«U<sub>max</sub>(14) =20»**

$$R = \text{Int}(\text{Sqr}(N))$$



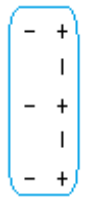
$$U_{\max}(R) = 2 * R * (R - 1)$$



$$\text{Int}((N - R^2) / R)$$

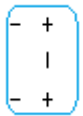
$$\text{Int}((N - R^2) / R) = \text{Int}((14 - 3^2) / 3) = 1$$

$$R + R - 1 = 2 * R - 1$$



«**N=3, U = 5**»

$$(N \text{ Mod } R) + (N \text{ Mod } R) - 1 = 2 * (N \text{ Mod } R) - 1 = 2 + 2 - 1 = 3$$



«**N=2, U = 3**»



$$R = \text{Int}(\text{Sqr}(N))$$

$$U = 2 * R * (R - 1) \text{ 'Square part}$$

$$U = U + \text{Int}((N - R^2) / R) * (2 * R - 1) \text{ 'Rectangular part}$$

$$\text{If } N \text{ Mod } R > 0 \text{ Then } U = U + 2 * (N \text{ Mod } R) - 1 \text{ 'Remainder part}$$

## 2.7. Comparison of multivariate methods

Evaluation	Original	ADZP	Correlation M.	P.C.A.	Factor A.	Quant.-III	Cluster
Seq. dist.	7.318	11.657	3.821	8.616	11.473	11.615	4.605
Ref. dist.	3.523	5.897	.506	1.712	.291	.018	.212
Seq. correl.	-.053	.495	-.467	.231	.435	.521	-.375
Ref. correl.	-.123	.587	-.602	.305	.442	.625	-.446
Mean contact	1.309	1.643	1.575	1.660	1.531	1.667	1.540
Norm. contact	.677	.850	.814	.858	.791	.862	.796

Table 19